DOI: 10.15199/17.2023.9.2

A comparative study of one-dimensional models for stratified thermal energy storage

Badanie porównawcze jednowymiarowych modeli warstwowych magazynowania energii cieplnej

Andrzej J. Osiadacz, Łukasz Kotyński, Małgorzata Kwestarz*)

Key words: *– thermal Energy, – numerical simulation, – mathematical modeling, – district heating networks, – temperature distribution.*

Abstract
In the article, a comparative analysis of one-dimensional dynamic models describing Thermal Energy Storages is carried out. A model based on energy balance was compared with a model of conductivity and convection and with a convection model. For each of them, the effect of time on the temperature distribution in the tank was examined and temperature distributions in the tank obtained for the analysed models were compared. Also, the effect of the number of nodes (discretization intervals) on the shape of the stratification curve was studied and the impact of the flow rate of the supplied hot water on the stratification curve was examined.

Słowa kluczowe: *energia cieplna, numeryczna symulacja, matematyczne modelowanie, dystrybucyjne sieci ciepłownicze, rozkład temperatury.*

Streszczenie
W artykule dokonano analizy porównawczej jednowymiarowych dynamicznych modeli opisujących Thermal Energy Storages. Porównywano model oparty na bilansie energii z modelem przewodzenia i konwekcji oraz modelem konwekcyjnym. Dla każdego modelu badano wpływ czasu na rozkład temperatury w zasobniku, porównywano rozkłady temperatury w zasobniku otrzymane dla analizowanych modeli a także badano wpływ liczby węzłów (przedziałów dyskretyzacji) na kształt krzywej stratyfikacji. Badano także wpływ natężenia przepływu dostarczanej wody gorącej na krzywą stratyfikacji.

1. Introduction

There is a growing interest in using hot water tanks as energy storage. The use of non-pressurized tanks is the simplest and cheapest way of short-term heat accumulation. The conducted research concerns issues related to the storage and use of energy in the form of heat used in power plants, heating plants and combined heat and power plants, as well as in solar systems. In municipal heating systems, there are significant fluctuations in the demand for heat power from consumers, both in the heating season and in the summer season. These fluctuations are analysed on a 24-hour basis. In the heating season, they are associated with rapid changes in the weather conditions, such as a decrease or increase in outdoor air temperature, insolation and wind speed, while in the summer season – mainly with the changing demand for hot utility water from consumers.

Fluctuations in the demand for heat power from consumers cause significant problems in operation, which force frequent changes in the heat power of boilers in heat sources, i.e. both heating plants and combined heat and power plants, which in turn affects the decrease in energy generation efficiency. If the district heating system is supplied from a combined heat and power plant, these problems are even greater as these fluctuations in heat demand significantly impede the stable production of electricity with high overall efficiency. Only due to the above problems, the introduction of additional heat capacity in the form of a heat accumulator into the district heating system seems to be highly reasonable. Properly located and properly designed heat accumulators bring significant economic and ecological effects in municipal heating systems.

Heat accumulators are usually vertical, cylindrical, above-ground, thermally insulated steel tanks. Depending on the location and purpose, they reach a height of several dozen centimetres to several dozen meters, with a capacity of up to tens of thousands of cubic meters. To increase their energy efficiency, we use the phenomenon of thermal stratification, which occurs in result of different densities of cold and hot water. A transition zone – called the thermocline – is formed between the layer of cold and hot water. In non-pressurized tanks, the temperature difference between hot and cold water ranges from 30 to 45°C.

Simulation research or optimization of heating network operation parameters requires the knowledge of both the dynamic model of water flow in the heat pipeline and the mathematical model of the storage tank.

The change in water temperature in the network depends, among others, on the water exchange process in the storage tank/s.

Thermal storage tanks also effectively improve the efficiency of solar systems by increasing the efficiency of solar collectors and by increasing the temperature of the water supplied for loading. In order to properly select the geometric dimensions, location (in the case of dispersed tanks) and operating parameters, it is necessary to study the phenomenon of thermal stratification that occurs in the tank. Thermal stratification is a complex phenomenon resulting from heat transfer and fluid dynamics. Mathematical modelling of the processes taking place in tanks is a complex task, but the simulation results are very helpful both in the process of designing new and optimizing the parameters of existing storage tanks. Depending on the desired accuracy of the description of the phenomenon,

* **Andrzej J. Osiadacz** – andrzej.osiadacz@pw.edu.pl, **Łukasz Kotyński** – lukasz.kotynski@pw.edu.pl , **Małgorzata Kwestarz** – malgorzata.kwestarz@pw.edu.pl, a District Heating and Gas Systems Department, Warsaw University of Technology, Nowowiejska Street 20, 00-653 Warsaw, Poland

various models of varying complexity are used, from detailed three-dimensional models to simple one-dimensional mathematical models. The three and two dimensional models are based on the fundamental equations of fluid dynamics: mass conservation or continuity, momentum and energy. By supplying them with state equation we are able to describe the thermal and hydrodynamic behaviour of thermal energy tank. Computer simulation using two – and three-dimensional models allows for a thorough analysis of the behaviour of thermal systems in various conditions. Very accurate models depend on empirical coefficients only to a limited extent, which is their great advantage. They can be a source of data used in the simulation process based on simpler models.

2. Literature review

The principle of operation of stratified storage tanks is based on the natural process of stratification. When a hot water tank without an external flow is exposed to ambient temperature during cooling, a process of thermal stratification of the water occurs. Cold water accumulates at the bottom while hot water rises to the top of the tank. This phenomenon occurs even when the water in the tank initially has the same temperature. This is due to the fact that before releasing heat to the environment, the wall of the tank cools the thin vertical layer of water adjacent to it. Part of the heat is transferred by diffusion towards the tank axis. A layer of cooler water with a higher density than its environment drops towards the bottom of the reservoir.

Thermal stratification is characterized by structural instability. There are several factors involved in stratification loss in thermal fluid storage tanks, such as

- heat loss to the environment through the tank shell,
- heat conduction from the hot layer to the cold layer,
- mixing caused by input/output streams during charging or discharging phases.
- vertical conduction in the tank wall, which, together with heat losses to the environment, creates convection currents that favour mixing.

Mathematical models of heat accumulators have been analysed in many publications.

In [11], a study of thermally stratified tanks for hot and cold-water storage applications, established a two – dimensional model, based on equations of mass conservation, amount of movement and energy to represent thermal transport process in the tank. The results obtained were compared with available experimental results and also with one-dimensional analytical model.

In the case of one-dimensional model, it is assumed that the flow inside the tank is one-dimensional and it is valid only when the inlet to the tank has a well-designed diffuser, and when the temperature gradient in radial direction is small. It means the convection currents due to the tank wall conduction can be neglected [9].

In the case of one-dimensional models, there are several types of models with varying degrees of complexity. In [1, 5-8], a number of models of differing complexity have been developed to account for thermal stratification in liquid storage tanks.

In the multi-node approach [1, 2, 6], the tank is modelled as n fully mixed volume segments (nodes). The degree of stratification is determined by the choice of n. Higher values of n result in greater stratification. If n=1, the tank is modelled as a fully mixed tank and no stratification effects are possible.

Energy balance for the i-th node is the following:

$$M_i C_f \frac{dT_i}{dt} = \alpha_i \dot{m}_{heat} C_f (T_{heat} - T_i) + \beta_i \dot{m}_{load} C_f (T_{mains} - T_i) + \delta_i \gamma_i C_f (T_{i-1} - T_i) + (1-\delta_i)\gamma_i C_f (T_i - T_{i+1}) + \varepsilon \dot{Q}_{aux,1} - (1-\epsilon) UA_{fl,i}(T_i - T_{fl}) - UA_i (T_i - T_{env}) \quad (1)$$

where:

α_i=1 if fluid from heat source enters node i, 0 otherwise

β_i=1 if fluid returning from load enters node i, 0 otherwise

$$\gamma_i = \dot{m}_{heat} \sum_{j=1}^{i-t} \alpha_j - \dot{m}_{load} \sum_{j=i+1}^{N} \beta_j$$

$$\delta_i = \begin{cases} 1, & if\ \gamma_i > 0 \\ 0, & if\ \gamma_i \le 0 \end{cases}$$

ε=1, if auxiliary is ON, 0 otherwise

Thus, we obtain n ordinary differential equations, which, when transformed into the differential form, create a system of linear algebraic equations. Once these equations are solved, the temperature values in each node at discrete time moments are obtained.

The plug flow model simulates the behaviour of a temperature-stratified storage tank using a variable number of variable size segments [1, 2, 7]. The number of segments and their volumes cannot be controlled: they vary, depending primarily on the tank volume, the net (heat source plus load) flow, and the simulation time step.

The mathematical model of plume entrainment, described in [8], is based on energy and mass balances. The storage tank is modelled as having two separate sections: the plume or stream region and the rest of the tank. These regions will be referred to as the stream (S) and the tank (T) regions. A one-dimensional model is formulated using the following equations:

Energy balances for the stream:

$$C_f \frac{\delta(\dot{m}_S T_S)}{\delta x} = C_f T_T \frac{\delta \dot{m}_S}{\delta x} \quad (2)$$

and for the tank

$$\rho_f A C_f \frac{\delta T_T}{\delta t} = -C_f \frac{\delta(\dot{m}_T T_T)}{\delta x} + C_f T_T \frac{\delta \dot{m}_T}{\delta x} + k_f A \frac{\delta^2 T_T}{\delta x^2} - U_T P_T (T_T - T_{env}) \quad (3)$$

Conservation of mass requires that:

$$\frac{\delta \dot{m}_S}{\delta x} = \frac{\delta \dot{m}_T}{\delta x} \quad (4)$$

To calculate $\dot{m}_s(x,t)$,$\dot{m}_T(x,t)$,$T_s(x,t)$and $T_T(x,t)$, one additional equation is needed to describe how mass from the tank is entrained into the falling stream.

The following equation is used:

$$\frac{\delta \dot{m}_S}{\delta x} = C \frac{\dot{m}_{heat}}{D} \quad (5)$$

Numerical values for C under specific conditions can be obtained from the theoretical work summarized by Schlichting and the experimental work of Hill [27]. In this model, C is assumed to be 0.32.

There are numerous papers on dynamic TES simulation using various one-dimensional mathematical models.

In [16], the model equations for the storage tank was derived based on the energy balance of the given system. Several assumptions were made in order to simplify the model. The tank was assumed to be adiabatic over the time period of interest, with no mass flow of stored water entering and leaving the tank. The fluids were considered incompressible and at a sufficiently high pressure so that no change in phase occurs. Finally, mixing between the stratified layers due to buoyancy was neglected. Conservation of energy was used in [17] to derive a system of n ordinary differential equations that can be solved numerically, yielding the temperature stratification in the storage tank as a function of time. The tank is assumed to be ideally stratified at the coil inlet height. In [18], a one-dimensional model was represented as a single partial differential equation, which was generated from a spatially dependent energy balance on the tank. Because the storage medium for thermocline systems are generally liquids assumed to be incompressible and because the thermocline tank is maintained at full volume, a mass balance is not required. Because heat transfer from one node to another can occur by diffusion, conduction, or axial mixing due to turbulent flow, the term (which has units of thermal conductivity) was used as a lumped parameter which represents the combined effect of these modes of heat transfer. In [19], the following has been assumed to simplify the mathematical modelling:

- the plug flow is established over the tank, i.e. only the uniform bulk flow at velocity v prevails without any mixing or disturbances,
- heat transfer to surroundings is negligible,
- thermos-physical properties are constant, and
- the inlet temperature and mass flow rate are kept constant.

Under these simplifications the transient temperature in the tank is governed by the following one-dimensional energy equation. In [20], a mathematical model of the heat transfer process in the tank with thermal stratification is based on the following assumptions:

- the tank has a cylindrical shape with a vertical axis,
- temperature of the liquid in the tank changes only in the axial direction,
- thermal properties of the liquid are constant in time and location--independent,

In [6], a one-dimensional model to describe the mass temperature is used to analyse the thermal dynamics in the TES. The tank is divided into N fixed layers with a thickness of Δx (m). The mass temperature of one layer is assumed to be homogeneous. Based on the conservation of energy, the balance equation is formulated for each node. In [22], an energy balance equation is formulated based on the energy conservation law. Thermal properties of the liquid are constant in time and location. The tank is divided into N fixed layers with a thickness of Δx (m). The mass temperature of one layer is assumed to be homogeneous. A balance equation is formulated for each node based on the conservation of energy.

In [23], the modelling approach used is based on the law of conservation of energy and it has been borrowed from [18]. The mathematical representation of the tank is constituted by nonlinear dynamic equations, where the thermal conductivity of water is neglected as its effect on the general system performance is insignificant [13].

2. Analyzed problems

The aim of this paper is to use numerical modelling techniques for simulating the transient one-dimensional fluid dynamics within the thermal energy storage tanks, with the view of improving thermal stratification and the overall efficiency of the system. More specifically, this paper will develop one-dimensional models to quantify the level of thermal stratification in a tank and to investigate the effects of tank ratio, inlet/outlet flow rate and inlet outlet position on the level of thermal stratification. It is believed that the outcome of this investigation will provide insight into the effects that these parameters have on thermal energy storage, which will be crucial for the maximization of the efficiency of a hot water storage tank.

The main objective of the research carried out was to analyse and compare the results of a dynamic simulation of a thermal heat storage tank using different mathematical models of the tank.

2.1. General energy equation

The balance law applied to a thermal system becomes an energy balance:

$$\frac{dE(t)}{dt} = \sum_i Q_i(t) \tag{6}$$

Where E [J] is the thermal energy, and $Q_i \left[\frac{J}{s}\right]$ is energy inflow node i. The energy E is assumed to be proportional to the temperature and the mass or volume:

$$\mathrm{E} = C_w mT = c\rho VT \tag{7}$$

Where T [K] is temperature, $C_w \left[\frac{J}{kg\,K}\right]$ is specific heat capacity, m [kg] is mass, V [m^3] volume, $\rho \left[\frac{kg}{m^3}\right]$ is density.

To analyse the thermal dynamics in TES, we use the one-dimensional model. The tank is divided into N fixed nodes with a thickness of $\Delta x(m)$. The mass temperature of one node is assumed to be homogeneous with the value of $T_i\,(t), i=1,2,\ldots,N$. For the i node, the delivered heat power consists of the heat conduction from the upper node $\dot{Q}_1$, the heat conduction from the lower node $\dot{Q}_2$, the heat input by charging $\dot{Q}_3$, the heat input by discharging $\dot{Q}_4$ and the heat of loss $\dot{Q}_5$.

Based on the energy balance equation (1), for the i node we have:

$$C_w m_i \frac{dT_i}{dt} = \dot{Q}_{1,i} + \dot{Q}_{2,i} + \dot{Q}_{3,i} + \dot{Q}_{4,i} + \dot{Q}_{5,i} \tag{8}$$

After discretization, equation (8) can be written in the following form:

$$C_w m_i (T_i^{n+1} - T_i^n) = (\dot{Q}_{1,i} + \dot{Q}_{2,i} + \dot{Q}_{3,i} + \dot{Q}_{4,i} + \dot{Q}_{5,i})\Delta t \tag{9}$$

where:

$$\dot{Q}_{1,i} = KA_1(T_{i-1}^n - T_i^n)/\Delta x \tag{10}$$

$$\dot{Q}_{2,i} = KA_1(T_{i+1}^n - T_i^n)/\Delta x \tag{11}$$

$$\dot{Q}_{3,i} = C_w m_c(T_{i-1}^n - T_i^n) \tag{12}$$

$$\dot{Q}_{4,i} = C_w m_d(T_{i+1}^n - T_i^n) \tag{13}$$

$$\dot{Q}_{5,i} = KA_2(T_z^n - T_i^n) \tag{14}$$

where:

– m_i – mass flow in node i [kg],
– A_1– tank cross section area [m^2],
– A_2 – tank lateral area [m^2],
– K – overall heat transfer coefficient $\left[\frac{W}{m\,K}\right]$,
– $\dot{m}_c$ – charging mass flow rate $\left[\frac{kg}{s}\right]$,
– $\dot{m}_d$ – discharging mass flow rate $\left[\frac{kg}{s}\right]$,
– T – temperature [K],
– T_j^i – temperature, i-th time level, j-th node,

Subsequent temperature values in each node are calculated from the following:

for j=1

$$T_1^{i+1} = \left\{\frac{KA_1}{\Delta x c_w m_1}\left[\left(T_{zas} - T_1^i\right) + \left(T_2^i - T_1^i\right)\right] + T_1^i + \frac{1}{m_1}\left[\dot{m_c}\left(T_{zas} - T_1^i\right) + \dot{m_d}\left(T_2^i - T_1^i\right)\right] + \frac{KA_2}{c_w m_1}\left(T_{zew} - T_1^i\right)\right\}\Delta t \tag{15}$$

for j=2, …, N-1

$$T_j^{i+1} = \left\{\frac{KA_1}{\Delta x c_w m_j}\left[\left(T_{j-1}^i - T_j^i\right) + \left(T_{j+1}^i - T_j^i\right)\right] + T_j^i + \frac{1}{m_j}\left[\dot{m_c}\left(T_{j-1}^i - T_j^i\right) + \dot{m_d}\left(T_{j+1}^i - T_j^i\right)\right] + \frac{KA_2}{c_w m_j}\left(T_{zas} - T_j^i\right)\right\}\Delta t \tag{16}$$

for j=N

$$T_N^{i+1} = \left\{\frac{KA_1}{\Delta x c_w m_N}\left[\left(T_{N-1}^i - T_N^i\right) + \left(T_{zas} - T_N^i\right)\right] + T_N^i + \frac{1}{m_N}\left[\dot{m_c}\left(T_{N-1}^i - T_N^i\right) + \dot{m_d}\left(T_{zas} - T_N^i\right)\right] + \frac{KA_2}{c_w m_N}\left(T_{zew} - T_N^i\right)\right\}\Delta t \tag{17}$$

The following parameter values were adopted in the simulation tests:

m_i – 78 500 [kg],
A_1 – 78.5 [m^2],
A_2 – 31.4 [m^2],
K – 0.1 $\left[\frac{W}{m\,K}\right]$,
$\dot{m}_c$ – 5 $\left[\frac{kg}{s}\right]$,
$\dot{m}_d$ – 0 $\left[\frac{kg}{s}\right]$,
c_w – variable according to temperature,

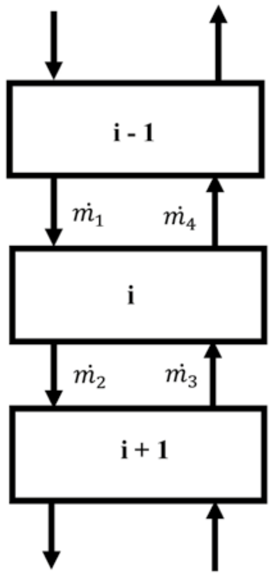

Fig. 1 Internal flows associated with node i

Rys.1 Wewnętrzne przepływy w i-tym węźle

T_{zas} – 383.15 [K],
T_{pow} – 323.15 [K],
T_{zew} – 263.15 [K],
T_1, T_i, T_N (initial temperature)– 343.15 [K],
H – height of the tank – 20 [m],
D – diameter of the tank – 10 [m],
N – number of nodes – 20.

Fig. 2 shows the results of simulation of temperature distribution in the tank, assuming the simulation time as a parameter. It can be seen

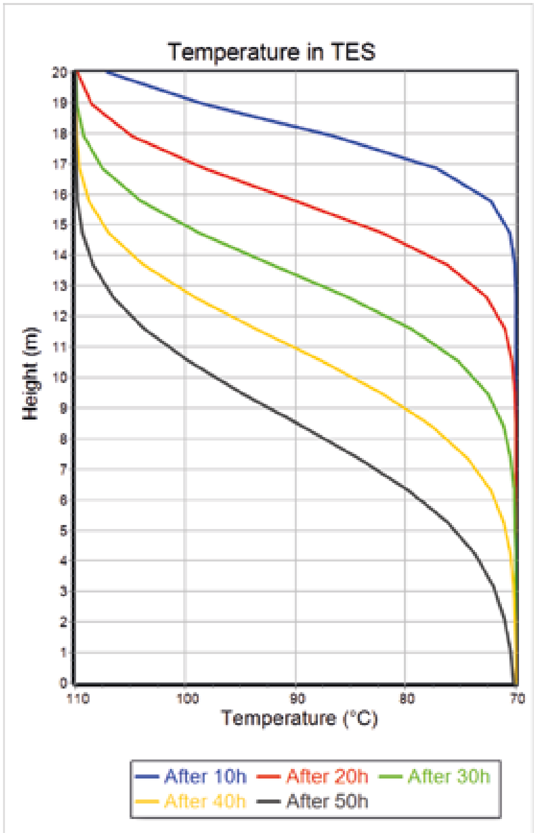

Fig. 2 Temperature change in TES for the balance equation

Rys.2 Zmiana temperatury w zasobniku wynikająca z równania energii

that the temperature distribution curve shifts over time, which means a gradual increase in the average temperature in the tank.

2.2. Transient heat by conduction and convection

Assuming that in a thermal tank not only the phenomenon of heat exchange but also convection occurs, the Fourier-Kirchhoff equation was adopted as a mathematical model to simulate the temperature distribution in the tank, depending on the boundary conditions and the initial condition:

$$\frac{\partial T}{\partial t} + \upsilon \frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \tag{18}$$

where:
υ – flow rate $\left[\frac{m}{s}\right]$
$\alpha = \frac{k}{c_p \varrho}$ – thermal diffusity $\left[\frac{m^2}{s}\right]$
k – thermal conductivity coefficient $\left[\frac{W}{m\,K}\right]$
ρ – density $\left[\frac{kg}{m^3}\right]$
c_p – specific heat $\left[\frac{J}{kg\,K}\right]$
T – temperature [K].
Equation (18) has the form of a differential scheme:

$$\frac{T_j^{n+1} - T_j^n}{\Delta t} + \frac{\upsilon}{2}\left[\frac{T_{j+1}^n - T_{j-1}^n}{2\Delta x} + \frac{T_{j+1}^{n+1} - T_{j-1}^{n+1}}{2\Delta x}\right] = \frac{\alpha}{2}\left[\frac{T_{j+1}^n - 2T_j^n + T_{j-1}^n}{(\Delta x)^2} + \frac{T_{j+1}^{n+1} - 2T_j^{n+1} + T_{j-1}^{n+1}}{(\Delta x)^2}\right] \tag{19}$$

In result of subsequent transformations:

$$\frac{T_j^{n+1} - T_j^n}{\Delta t} + \frac{\upsilon}{2}\left[\frac{T_{j+1}^n - T_{j-1}^n + T_{j+1}^{n+1} - T_{j-1}^{n+1}}{2\Delta x}\right] = \frac{\alpha}{2}\left[\frac{T_{j+1}^n - 2T_j^n + T_{j-1}^n + T_{j+1}^{n+1} - 2T_j^{n+1} + T_{j-1}^{n+1}}{(\Delta x)^2}\right] \tag{20}$$

$$T_j^{n+1} - T_j^n = -\overset{\alpha}{\frac{\upsilon\Delta t}{4\Delta x}}\left[T_{j+1}^n - T_{j-1}^n + T_{j+1}^{n+1} - T_{j-1}^{n+1}\right] + \overset{\beta}{\frac{\alpha\Delta t}{2(\Delta x)^2}} \left|T_{j+1}^n - 2T_j^n + T_{j-1}^n + T_{j+1}^{n+1} - 2T_j^{n+1} + T_{j-1}^{n+1}\right| \tag{21}$$

we obtain:

$$T_j^{n+1} + \alpha T_{j+1}^{n+1} - \alpha T_{j-1}^{n+1} - \beta T_{j+1}^{n+1} + \beta 2T_j^{n+1} - \beta T_{j-1}^{n+1} = T_j^n - \alpha T_{j+1}^n + \alpha T_{j-1}^n + \beta T_{j+1}^n - \beta 2T_j^n + \beta T_{j-1}^n \tag{22}$$

$$T_{j-1}^{n+1}(-\alpha - \beta) + T_j^{n+1}(1 + 2\beta) + T_{j+1}^{n+1}(\alpha - \beta) = T_{j-1}^n(\alpha + \beta) + T_j^n(1 - 2\beta) + T_{j+1}^n(\beta - \alpha) \tag{23}$$

Equation (23) for n=10 nodes has the following form:

$$\underline{A}\,\underline{x} = \underline{B}\,\underline{c} \tag{24}$$

where:

$$\underline{A} = \begin{array}{c} \\ 2\\3\\4\\5\\6\\7\\8\\9 \end{array} \begin{array}{c} \begin{array}{cccccccc} 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{array} \\ \begin{bmatrix} (1+2\beta) & (\alpha-\beta) & & & & & & \\ (-\alpha-\beta) & (1+2\beta) & (\alpha-\beta) & & & & & \\ & \ddots & \ddots & \ddots & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & (-\alpha-\beta) & (1+2\beta) & (\alpha-\beta) \\ & & & & & & (-\alpha-\beta) & (1+2\beta) \end{bmatrix} \end{array}$$

$$\underline{x} = \begin{bmatrix} T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \\ T_9 \end{bmatrix} \qquad \underline{c} = \begin{bmatrix} T_2^n + T_1^n(\alpha+\beta) \\ T_3^n \\ T_4^n \\ T_5^n \\ T_6^n \\ T_7^n \\ T_8^n \\ T_9^n - T_{10}^n(\alpha-\beta) \end{bmatrix}$$

$$\underline{B} = \begin{array}{cc} & \begin{array}{cccccccc} 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{array} \\ \begin{array}{c} 2\\3\\4\\5\\6\\7\\8\\9 \end{array} & \begin{bmatrix} (1-2\beta) & (\beta-\alpha) & & & & & & \\ (\alpha+\beta) & (1-2\beta) & (\beta-\alpha) & & & & & \\ & \ddots & \ddots & \ddots & & & & \\ & & \ddots & \ddots & \ddots & & & \\ & & & \ddots & \ddots & \ddots & & \\ & & & & \ddots & \ddots & \ddots & \\ & & & & & (\alpha+\beta) & (1-2\beta) & (\beta-\alpha) \\ & & & & & & (\alpha+\beta) & (1-2\beta) \end{bmatrix} \end{array}$$

The coefficient matrices A and B are tridiagonal.

The system of equations (24) was solved with the following values:

υ – 2,5E-4 $\left[\frac{m}{s}\right]$

k – variable according to temperature $\left[\frac{W}{m\,K}\right]$

ρ – variable according to temperature $\left[\frac{kg}{m^3}\right]$

c_p – variable according to temperature $\left[\frac{J}{kg\,K}\right]$

T_1 – 383.15 [K],

T_{10} – 323.15 [K],

T_1, T_i, T_N (initial temperature) – 343.15 [K].

The results obtained practically coincide with the results obtained in the first variant, which confirms the correctness of both formulations.

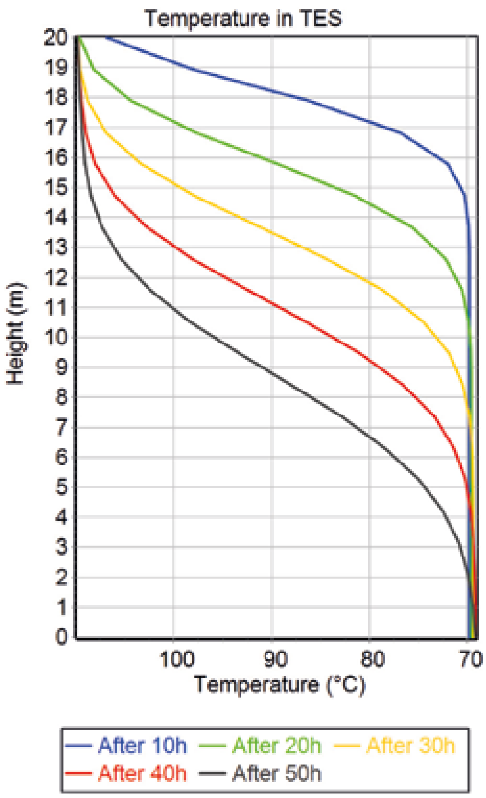

Fig. 3 Temperature change in TES for the conduction and convection equation

Rys.3 Zmiana temperatury w zasobniku wynikająca z rownania przewodzenia i konwekcji

2.3. Convection model

In order to find out about the effect of diffusion on temperature distribution in the tank, a mathematical model of the convection process was adopted for the simulation, and the results of the simulation were compared with the results of the simulation using the convection-diffusion model:

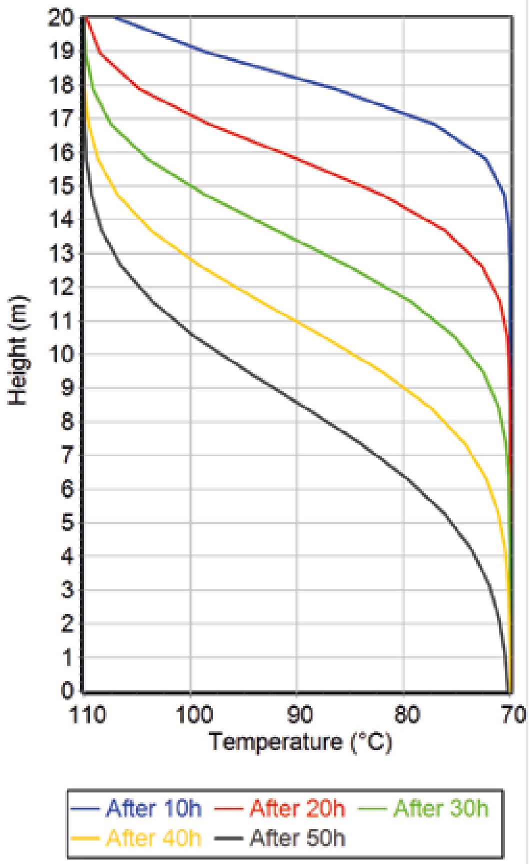

Fig. 4 Temperature change in TES for the convection equation

Rys.4 Zmiana temperatury w zasobniku wynikająca z równania konwekcji

$$\frac{\partial T}{\partial t} + \upsilon\frac{\partial T}{\partial x} = 0 \tag{25}$$

where: υ – flow rate $\left[\frac{m}{s}\right]$

In time we use forward derivative:

$$\frac{\partial T}{\partial t} = \frac{T_j^{n+1} - T_j^n}{\Delta t}$$

In space, we use centred derivative:

$$\frac{\partial T}{\partial x} = \frac{T_{j+1}^n - T_{j-1}^n}{2\Delta x}$$

Ultimately, we obtain:

$$\frac{T_j^{n+1} - T_j^n}{\Delta t} + \upsilon\frac{T_{j+1}^n - T_{j-1}^n}{2\Delta x} = 0 \tag{26}$$

For T_j^{n+1} we have:

$$T_j^{n+1} = T_j^n - \frac{C}{2}\left(T_{j+1}^n - T_{j-1}^n\right) \tag{27}$$

Where: $C = \upsilon\,\frac{\Delta t}{\Delta x}$ is called the Courant –Friedrichs-Lewy number.

Condition for stability $\Delta t \le \frac{\Delta x}{\upsilon}$

The calculations were made assuming the same values as for the convection-diffusion model.

3. Model analysis

Fig. 5 presents temperature changes in TES for the same input parameters for the three models described above, assuming the simulation time of 50h. The first analysis concerned the change in temperature when the tank was divided into 20 nodes.

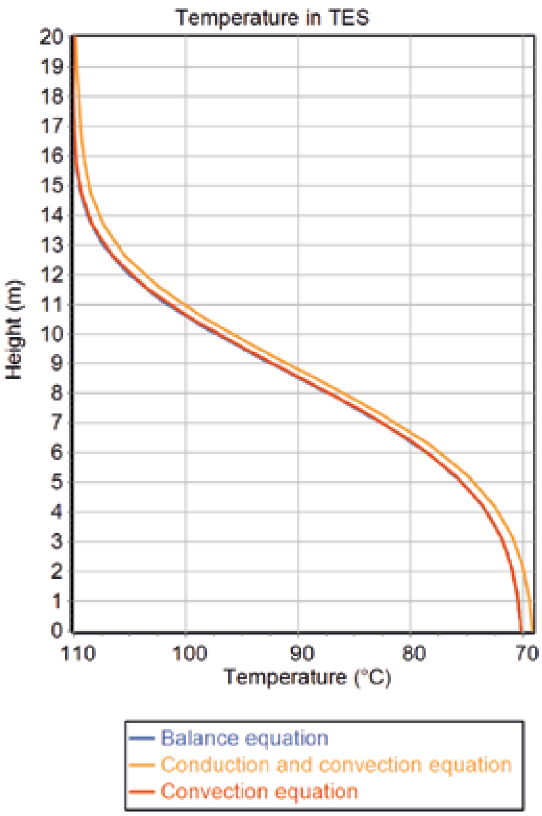

Fig. 5 Temperature change in TES for different equations (for 20 nodes)
Rys.5 Zmiana temperatury w zasobniku dla różnych modeli (dla 20 węzłów)

The results for the balance equation and for the convection model give almost identical results (in the graph, the lines representing the results for these models coincide). Very small differences between the balance and convection model on the one hand, and the convection-diffusion model on the other hand, reveal a very small effect of the diffusion phenomenon on the TES temperature distribution for the assumed tank discretization.

The simulation results for N=40 are shown in Fig. 6. The increase of the number of nodes affected the accuracy of the results obtained and confirmed that the results of the balance equation are convergent with the results of the convection equation. The difference between models I and II and model III (convective-diffusion) increased, which means a greater impact of the diffusion phenomenon on the temperature distribution at a smaller distance between the nodes.

For the analysed tank with a height of 20 m, the division into 20 nodes results in the creation of homogeneous temperature zones with a height of 1 m. It should be remembered that the bigger the number of nodes, the higher the accuracy of the results (Fig. 7). The division into 60 nodes creates zones with a height of approx. 33 cm. When divided into 80 nodes (zones of 25 cm each), the thermocline zone between 8 and 9 meters of the tank height is clearly distinguished.

The tank models discussed above were also analysed in terms of response to an increase in the stream [kg/s] of hot water flowing into the reservoir. The higher the stream, the faster the temperature should rise in the tank and the thermocline should form at the lower levels of the reservoir. The correct functioning of the presented models is confirmed by the graphs in Fig. 8.

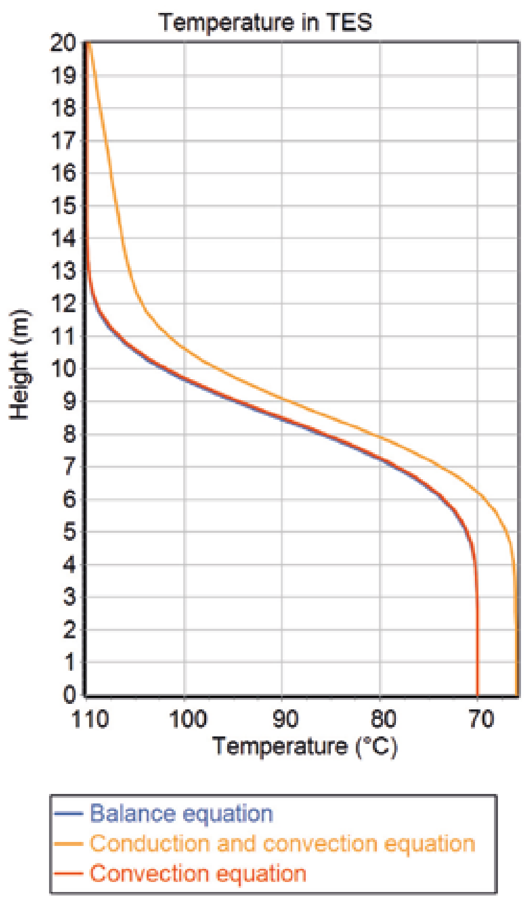

Fig. 6 Temperature change in TES for different equations (for 40 nodes)
Rys.6 Zmiana temperatury w zasobniku dla różnych modeli (dla 40 węzłów)

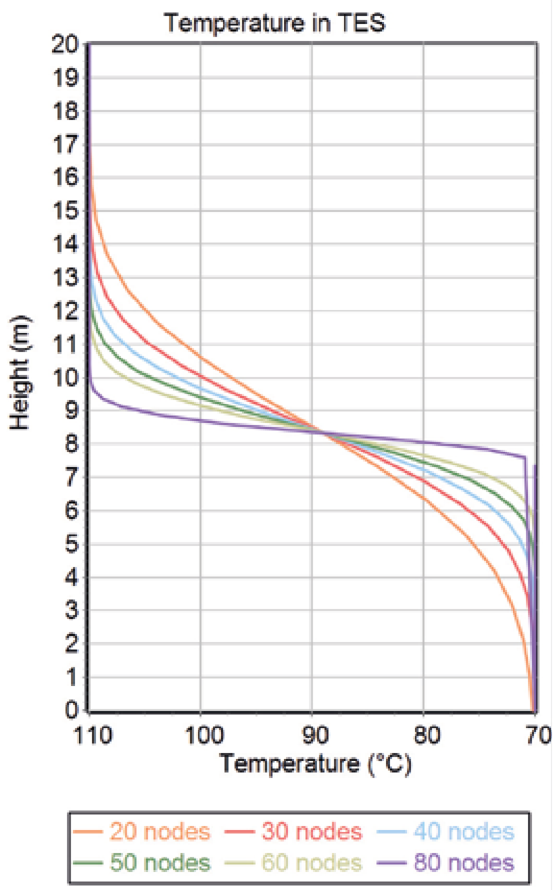

Fig. 7 Temperature change in TES for different nodes count
Rys.7 Zmiana temperatury w zasobniku dla różnej liczby węzłów

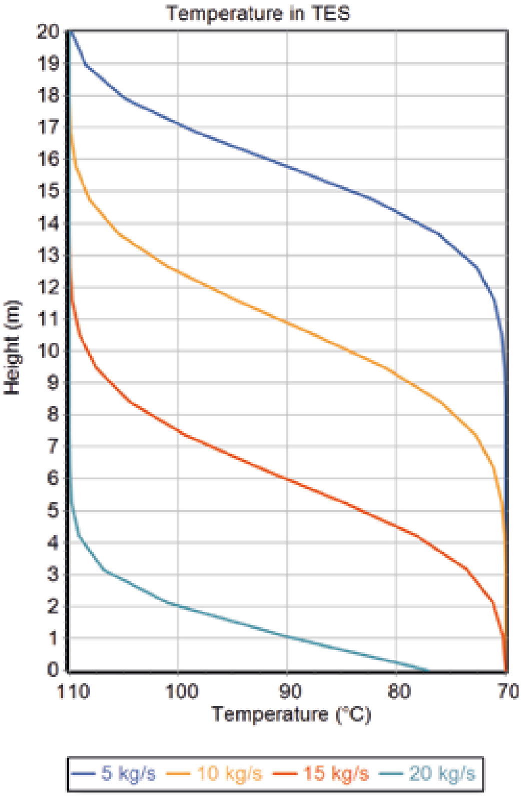

Fig. 8 Temperature change in TES for different charging flow rates
Rys.8 Zmiana temperatury w zasobniku dla różnych wartości przepływu.

4. Final conclusions

The research results presented in the article confirm the correctness of the formulated TES models. The one-dimensional model is naturally a compromise between the accuracy of the description of the phenomenon and the efficiency of the numerical solution. However, it should be remembered that the more complex the mathematical model used for the simulation, the more input data is necessary. This, in turn, requires a very good theoretical knowledge of the simulated phenomenon. It should be noted that computer simulations using 2D and 3D models, even for short real-time periods, take a long time. This means that in most cases these models are not suitable for use in simulation or optimization programs. The time of tank simulation calculations significantly extends the entire calculation process. It should be noted that in some cases, the differences in the results obtained for 1D and 2D are small, although the calculation time is incomparable. The one-dimensional TES model is successfully used in the process of dynamic simulation of a district heating network. ■

LITERATURA

[1] Bastida Hector, Carlos E. Ugalde-Loo, Muditha Abeysekera, Meysam Qadrdan, Jianzhong Wu, Nick Jenkins.2018. "Dynamic modelling and control of thermal energy storage" 10th Int. Conference on Applied Energy (ICAE 2018) Hong Kong.

[2] Chin Mi-Soo et. Al.2003."Numerical and experimental study on the design of stratified thermal storage system" *Applied Thermal Engineering.*

[3] Duffie J.A., W.A. 1991."Beckman, Solar engineering of thermal process, 2nd edition". *Wiley Interscience*, New York.

[4] Hill B.J.1972."Measurements of local entrainment rate in the initial region of axisymmetric turbulent air jet" *J. Fluid Mech*. (51) :773-779.

[5] Ievers Simon, Wenxian Lin. 2009."Numerical simulation of three dimensional flow dynamics a hot water storage tank"*Applied Energy* (86):2604-2614.

[6] Ismail K.A.R., J.F.B. Leak, M.A. Zanardi. 1996."A two dimensional model for liquid storage tanks"*Transactions on Engineering Science*, Vol.12, , WIT Press.

[7] Klein S.A. et. al., 1990."TRNSYS 13.1 User`s manual, Engineering Experiment Station Report 38-13". *Solar Energy Laboratory*, University of Wisconsin-Madison.

[8] Klein S.A.1976."A design procedure for solar heating systems" Ph.D. Thesis, Department of Chemical Engineering, University of Wisconsin-Madison.

[9] Kuhn J.K., G.F. Van Fuchs, A.P. Zob.1980." Developing and upgrading of solar system thermal energy storage simulation models, Draft Report for DOE, Boeing Computer Services Company, August 31 .

[10] Kusyi Oleg, Antoine Dalibard, 2007."Different methods to model thermal stratification in storage tanks" SOINET Ph.D. Course, , DTU.

[11] Lu Shuai, Wei Gu, Shuai Yao, Guangsheng Pan, Ke Meng, Zhao Yang Dong, 2020."Dispatch of Integrated Energy Systems considering thermal dynamics of thermal energy storage" IEEE Power & Energy Society General Meeting, pp. 1-25.

[12] Morales Sandoral Daniel A., Iwan De La Cruz Loredo, Hector Bastida, Jordan J.R. Badman, Carlos E. Ugalde-lOO, Desing and verification of an effective state of charge estimator for thermal energy storage, https://ietresearch.onlinelibrary.wiley.com/doi/10.1049/stg2.12024

[13] Moreira G.A.A., A. Barbasa, A.L. Viana, R.M. Valle.2020. "Three dimensional numerical simulation of a vertical thermal tank," E, Engenharia Termica (Thermal Engineering), Vol. 19, No 1 :96-103, June 2020.

[14] Nash Austin L., Apurra Badithela, Neera Jain,2017." Dynamic modelling of a sensible thermal energy storage tank with an immersed coil heat exchanger under three operations modes" *Applied Energy* 195 : 877-889.

[15] Neupaner Krzysztof, Krzysztof Kupiec.2017."Heat transfer during storage of hot liquid in the tank" *Technical Transcriptions*, 4 :1-12.

[16] Oliveski Rejana de Cesaro, Arno Krenzinger.2003." Cooling of cylindrical vertical tanks submitted to natural internal convection" Int. *Journal of Heat and Mass transfer* 46 :2015-2026.

[17] Oppel F.J., A.J. Ghajar, P.M. 1986."Moretti, Computer simulation of stratified heat storage" *Applied Energy* 23 : 205-224.

[18] Pate R.A., 1977."A thermal energy storage tank model for solar heating" Ph.D. Thesis, Utah State University, Logan, UT.

[19] Phillips W.F., R. N. Dave, 1982."Effects of stratification on the performance of liquid based solar heating systems" *Solar Energy,* 29 :111-120.

[20] Phillips W.F., R.A. Pate, 1977."Mass and energy transfer in a hot liquid energy storage system, Proceedings of the American Section of the International Solar Energy Society" Orlando, FA,.

[21] Powell Kody M., Thomas F. Edgar, 2013."An adaptive grid model for dynamic simulation of thermocline thermal energy storage systems" *Energy Conversion and Management* 76 : 865-873.

[22] Rahman Aowabin, Nelson Fumo, Amanda D. Smith, 2015."Simplified Modelling of Thermal Storage Tank for distributed energy heat recovery applications, Proceeding of the ASME" USA.

[23] Sharp M.K., R.I. Loehrke, 1979."Stratified thermal storage in residential solar applications" J. *Energy* 3 (2) :106-113.

[24] Schlichting H 1968." Boundary layer theory, 6th edition, McGraw-Hill, New York.

[25] Wuestling M.D., S.A. Klein, J.A. Duffie, 1985 ."Promising control alternatives for solar water heating systems,"*ASME Journal Solar Energy Eng*., 107 :215-221.

[26] Zurigat Y.H., K.J. Maloney, A.J. Ghajar, 1989."A comparison study of one-dimensional models for stratified thermal storage tanks" Trans. ASME, 111:204-210.

[27] Yoo Hoseon, EE-Tong PAK, 1996."Analytical soluton to a one-dimensional finite-domain model for stratified thermal storage tanks" Solar Energy Vol.56, (4): 315-322.