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Water hummer in fluid networks

Uderzenie hydrauliczne w sieciach płynowych

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Key words: *gas networks, water networks, multiphase flow, numerical methods, mathematical models, computer simulation*

Abstract
The article discusses the problem of water hammer in fluid networks (gas, water supply and in pipelines transporting two-phase fluids. Mathematical models and selected computational algorithms are given

Słowa kluczowe: *sieci gazowe, sieci wodociągowe, przepływy dwufazowe, metody numeryczne, modele matematyczne, symulacja komputerowa*

Streszczenie
W artykule omówiono problem uderzenia hydraulicznego w sieciach płynowych (gazowych, wodociągowych oraz w rurociągach transportujących płyny dwufazowe. Podano modele matematyczne oraz wybrane algorytmy obliczeniowe

Introduction

Water hammer (or hydraulic shock) is the momentary increase in pressure, which occurs in a water system/gas system or multiphase system, when there is a sudden change of direction or velocity of the fluid. When a rapidly closed valve suddenly stops fluid flowing in a pipeline, pressure energy is transferred to the valve and pipe wall. Shock waves are set up within the system. Pressure waves travel backward until encountering the next solid obstacle, then forward, then back again. The pressure wave's velocity is equal to the speed of the sound; therefore it "bangs" as it travels back and forth, until dissipated by friction losses. Anyone who has lived in an older house is familiar with the "bang" that resounds through the pipes when a faucet is suddenly closed. This is an effect of water hammer.

A less severe form of hammer is called surge, a slow motion mass oscillation of fluid caused by internal pressure fluctuations in the system. This can be pictured as a slower "wave" of pressure building within the system. Both water fluid and surge are referred to as transient pressures. If not controlled, they both yield the same results: damage to pipes, fittings, and valves, causing leaks and shortening the life of the system. In the case of water networks ,neither the pipe nor the water will compress to absorb the shock.

Investigating the causes of water hammer

A water transport systems operating conditions are almost never at a steady state. Pressures and flows change continually as pumps start and stop, demand fluctuates and tank levels change. In addition to these normal events, unforeseen events as power outages and equipment malfunctions, can sharply change the operating conditions of a system. Any change in liquid flow rate, regardless of the rate or magnitude of change, requires that the liquid be accelerated or decelerated from its initial flow velocity. Rapid changes in flow rate require large forces that are seen as large pressures, which cause water hammer.

Entrained air or temperature changes of the water also can cause excess pressure in the water lines. Air trapped in the line will compress and will exert extra pressure on the water. Temperature changes will actually cause the water to expand or contract, also affecting pressure. The maximum pressures experienced in a piping system are frequently the result of vapor column separation, which is caused by the formation of void packets of vapor when pressure drops so low that the liquid boils or vaporizes. Damaging pressures can occur when these cavities collapse.

The causes of water hammer are varied. There are, however, four common events that typically induce large changes in pressure:

1. Pump startup can induce the rapid collapse of a void space that exists downstream from a starting pump. This generates high pressures.
2. Pump power failure can create a rapid change in flow, which causes a pressure upsurge on the suction side and a pressure downsurge on the discharge side. The downsurge is usually the major problem. The pressure on the discharge side reaches vapor pressure, resulting in vapor column separation.
3. Valve opening and closing is fundamentalto safe pipeline operation. Closing a valve at the downstream end of a pipeline creates

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a pressure wave that moves toward the reservoir. Closing a valve in less time than it takes for the pressure surge to travel to the end of the pipeline and back is called "sudden valve closure." Sudden valve closure will change velocity quickly and can result in a pressure surge. The pressure surge resulting from a sudden valve opening is usually not as excessive.

4. Improper operation or incorporation of surge protection devices can do more harm than good. An example is oversizing the surge relief valve or improperly selecting the vacuum breaker-air relief valve. Another example is to try to incorporate some means of preventing water hammer when it may not be a problem.

In order to take into account the effect of the fluid velocity upon the pressurehead during water-hammer phenomenon the fundamental differential equations describing this phenomenon take the form:

$$\frac{\partial H}{\partial t} + \frac{a}{c}\frac{\partial Q}{\partial x} = 0 \tag{1}$$

$$\frac{\partial Q}{\partial t} + ac\frac{\partial H}{\partial x} + RQ|Q| = 0 \tag{2}$$

where the partial differential equations (1) and (2) correspond to the continuity and momentum (dynamics), respectively. Besides, H is the piezometric head, a is the wave speed, $c = (gA/a)$, where g is the acceleration of gravity, A is the pipe cross-section, Q is the fluid flow and $R = f/2DA$, f is the friction factor (Darcy-Weisbach) and D is the inner pipe diameter. The subscripts x and t denote

space and time dimensions, respectively. Partial differentia equations (1) and (2), in conjunction with the equations related with the boundary conditions of specific devices, describe the phenomenon of wave propagation for a water hammer event.

Wave speed

For water, without presence of free air or gas, the more general equation to calculate the water hammer wave speed magnitude in one-dimensional flows is (Watters, 1984):

$$a^2 = \frac{K}{\rho\left(1 + \frac{D}{e}\frac{K}{E}\Psi\right)} \tag{3}$$

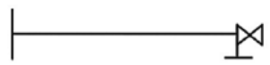
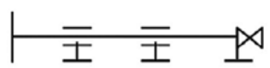
with a the wave speed, K he volumetric compressibility modulus of the liquid, ρ the liquid density, e the pipe wall thickness, E the pipe elasticity modulus (Young); ψ a factor related with the pipe supporting condition which can be calculated from general expressions (see Table 1) being the case 2 more conservative from an engineering point of view. Equation (3) supposes that:

- Pipe has a thin internal wall, condition which is met when D/e > 40 (Watters, 1984) or when D/e > 25 (Wylie and Streeter, 1978).
- Pipe remains full of water during the transient event; that is, no separation of the water column is generated, which means that at all times the pressure is greater than the vapour pressure.
- Water has small air content, so that the magnitude of the wave speed may be assumed constant.
- The pressure is uniform across any section of the pipe. It means that inertial forces associated with radial motion of the fluid are negligible (Skalak, 1955).

Equation (3) includes Poisson's ratio effect but neglects the motion and inertia of the pipe. This is acceptable for rigidly anchored pipe systems such as buried pipes or pipes with high density and stiffness, to name only a few. Examples include major transmission pipelines like water distribution systems, natural gas lines and pressurized and surcharged sewerage force mains. However, the motion and inertia of pipes can become important when pipes are inadequately restrained (unsupported, free-hanging pipes) or when the density and stiffness of the pipe is small.

Table 1: Expressions for ψ according to the pipe supporting conditions (Watters, 1984)

Tabela 1: Wyrażenia dla ψ zgodnie z warunkami podparcia rury

Case	Pipe supporting condition
1	Pipe anchored at the upstream end only
	
	$\Psi = [1/(1 + e/D)]\,[5/4 - u + 2(e/D)(1+u)(1 + e/D)]$
2	Pipe anchored against any axial movement
	
	$\Psi = [1/(1 + e/D)]\,[1 - u^2 + 2(e/D)(1 + u)(1 + e/D)]$
3	Case 2 plus longitudinal expansion joints along the pipeline
	
	$\Psi = [1/(1 + e/D)]\,[1 + 2(e/D)(1 + u)(1 + e/D)]$

Method of the characteristics

The Method of the Characteristics MOC is an Eulerian numerical scheme (Wood et al., 2005) very used for solving the equations which governing the transient flow because it works with a constant and, unlike other methodologies based on finite difference or finite element, it can easily model wave fronts generated by very fast transient flows. The essence of the method of characteristics is the successful replacement of a pair of partial differential equations by an equivalent set of ordinary differential equations. The development of the method begins by presuming that the pair of Eqs. 1 and 2 may be replaced by some linear combination of themselves. Using λ as a constant linear scale factor, sometimes called a Lagrange multiplier, one possible combination is:

$$\lambda\left(\frac{\partial Q}{\partial t} + ac\frac{\partial H}{\partial x} + RQ|Q|\right) + \left(\frac{\partial H}{\partial t} + \frac{a}{c}\frac{\partial Q}{\partial x}\right) = 0 \tag{4}$$

After regrouping terms and transformations, we have the following pair of ordinary differentia equations:

$$\frac{dQ}{dt} + ac\frac{dH}{dx} + RQ|Q| = 0 \tag{5}$$

$$\frac{dQ}{dt} - ac\frac{dH}{dx} + RQ|Q| = 0 \tag{6}$$

However, there are now special constraints imposed on the independent variables in each equation. Equation (5) is valid only when $\frac{dx}{dt}$ = +a. Similarly, equation (6) is valid only when $\frac{dx}{dt}$ t= – a.

Thus we have replaced two partial differential equations by two pairs of ordinary differential equations, and we must follow these rules which relate the independent variables x and t. The new form of eqs. 5 and 6 is now:

$$\frac{dQ}{dt} + ac\frac{dH}{dx} + RQ|Q| = 0 \quad \text{only when} \quad \frac{dx}{dt} = +a. \tag{7}$$

$$\frac{dQ}{dt} - ac\frac{dH}{dx} + RQ|Q| = 0 \quad \text{only when} \quad \frac{dx}{dt} = -a. \tag{8}$$

From the fact that special relations must be maintained between x and *t* in eqs. 7 and 8, the equations $\frac{dx}{dt} = +a$ and $\frac{dx}{dt} = -a$ have come to be called the *characteristics* of eqs. 7 and 8, hence the name of the analysis procedure. The characteristics equations can be presented graphically as shown in Fig. 1.

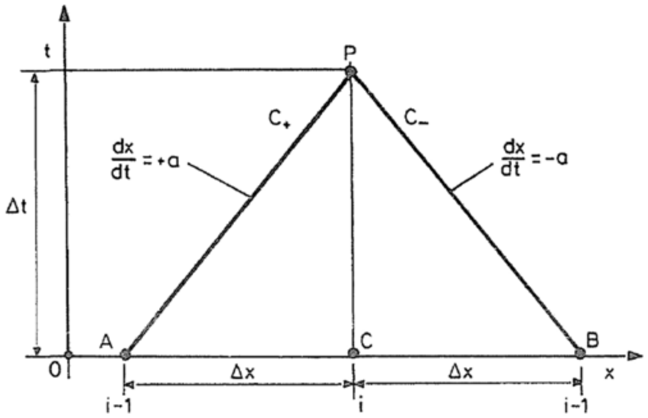


Fig.1 The x-t plane showing characteristics for eqs. 7 and 8
Rys.1 Graficzna prezentacja równań 7 oraz 8

For a constant value of pressure wave velocity a, lines *AP* and *BP* are straight lines.

MOC works converting the computational space x − time t grid (or rectangular mesh) in accordance with the Courant condition. It is useful for modelling the wave propagation phenomena in water distribution systems due to its facility for introducing the hydraulic behaviour of different devices and boundary conditions (valves, pumps, reservoirs, etc.). Among its main a dvantages it can be highlighted its ease of use, speed and explicit nature, which allows calculate the variables Q and H directly from previously known values (Chaudhry, 1979; Wylie and Streeter, 1978). The main disadvantage of the MOC is that it must fulfil with the Courant stability criterion that can limit the magnitude of the time step Δt common for the entire network. In order to get $C_n = 1$, some pipe initial properties can be modified (length and/or wave speed). Another way is to keep the initial conditions and apply numerical interpolations with risk of generating errors (numerical dissipation and dispersion) in the solution (Goldberg and Wylie, 1983). The MOC stability criterion states that (Watters, 1984):

$$C_n = a\frac{\Delta t}{\Delta x} \leq 1 \tag{9}$$

where Cn is the Courant number, Δt is the time step and Δx is the sub-section pipe length (Δx = L/N with L the pipe length and N the number of pipe sub−sections). In general, MOC gives exact numerical results when Cn = 1, otherwise, it generates erroneous results in the way of attenuations (when Cn < 1) or numerical instability (when Cn > 1).

For many years the Method of the Characteristics MOC has been used for solving the transient flow in pipe networks due to its numerical efficiency, computational accuracy, and programming simplicity. However, one difficulty that arises is the selection of an appropriate time step Δt to use for the analysis. The challenge of selecting a time step is made difficult in pipeline systems because to calculate head and discharge in many boundary conditions it is necessary that the time step be common to all pipes. Besides, MOC requires that the ratio of the distance step Δx to the time step Δt be equal to the wave speed a in each pipe, or that the Courant number Cn = a Δ t/Δx should ideally be equal to one. For most pipeline systems it is impossible to satisfy exactly the Courant requirement with a reasonable (and common) Δt because they have a variety of different pipes with a range of wave speeds and lengths (Karney and Ghidaoui, 1997). There are two strategies to deal with this problem. The first strategy is apply the method of the wave-speed adjustment MWSA where one of the pipeline properties is altered (usually wave speed) to satisfy exactly the Courant condition. The second strategy is interpolating between known grid points allowing Courant numbers less than one. At first glance the MWSA appears simpler because is non-dissipative and non-dispersive and in theory only consists in modify the value of the wave speed in a certain percentage to meet Cn = 1. Nevertheless, this procedure distorts the physical characteristics of the problem (Ghidaoui and Karney, 1994). In other words, changing a involves altering, in physical terms, the value of one or more of the parameters that are part of its formulation such as fluid density or the elastic modulus of the constituent element of the pipe. More clearly, the modification of a in numerical terms involves altering the initial physical conditions of the system, leading to a solution that may be correct in numerical terms (to meetCn), but incorrect in physical terms because the problem is solved using parameters with unreal magnitudes.

Sectioning for piping systems: method of wave-speed adjustment

In piping systems Δt must equal for all pipes. This involves a certain amount of care in its selection. It is quickly realized that (4) probably cannot be exactly fulfilled in most systems. In as much as the wave speed is probably not known with great accuracy, it may be permissible to adjust it slightly, so that integer N may be found. In equation form this can be expressed as (Wylie and Streeter, 1978):

$$\Delta t = \frac{L_j}{a_j\left(1 \pm \varnothing_j\right)N_j} \tag{10}$$

in which $\varnothing_j$ is a permissible variation in the wave speed in pipe j, always less than some maximum limit of say 0.15 or 15% (Wylie and Streeter, 1978). In general, a slight modification in wave speed is more preferable than any alteration in pipe length to satisfy the requirement of a common time step size.

Numerical interpolation

When MOC is applied with Cn < 1 some numerical interpolation must be applied in order to obtain Q and H for every pipe inner section. When the interpolation is applied on the x axis, some analytical expressions can be obtained for the state variables Q and H at interior nodes using numerical schemes with different interpolation orders. The most common numerical interpolation methods include

linear interpolation at a fixed time level, including both space line interpolation and reach−out in space interpolation, as well as interpolation at a fixed location, such as time line interpolation or reach−back in time interpolation (Karney and Ghidaoui, 1997). There is a tendency among8practitioners to think of interpolation as a numerical device with only numerical side effects. In general, all common interpolation procedures result in numerical dissipation and dispersion, and they considerably distort the original governing equations. The interpolation procedures effectively change the wave speed (Ghidaoui and Karney, 1994). In summary, interpolation fundamentally changes the physical problem and must be viewed as a nontrivial transformation of the governing equations. Because this topic is beyond the scope of this paper, more information will not be included here. In the following paragraphs, the main parameters of the wave speed in (3) will be briefly analyzed, showing their characteristic values. Compressibility is the property of a fluid to change its volume due to the pressure (Del Valle, 2010). For problems involving the effect of water hammer is necessary to take into account the compressibility of water, which is inversely proportional to its bulk modulus of elasticity and is defined mathematically as:

$$\varepsilon = \frac{-dP}{\left(\frac{dv}{v}\right)} \tag{11}$$

where v is the specific volume and P is pressure. The bulk modulus of elasticity K is:

$$K = \frac{-\left(\frac{dv}{v}\right)}{dP} \tag{12}$$

The equation (7) represents the relative change in a fluid volume per unit of applied pressure. The negative sign is because as the pressure increases, the volume decreases and vice versa. The ε units are the same for pressure. At a temperature of 20°C and atmospheric pressure (1 bar) the bulk modulus of water is $K = 2.07 \cdot 109$ Pa. The density of water is the weight of the water per its unit volume:

$$\rho = \frac{m}{V} \tag{13}$$

with ρ the density, m the fluid mass and V the fluid volume. The fluid density is function of pressure and temperaturę (especially in gases), it increases with increasing pressure and it decreases with major temperature. At atmospheric pressure and temperature of 4°C the water density is $\rho = 1000$ kg/m^3. The Young's elasticity modulus E is the relationship between the force increment and the unitary strain (Martínez and Azuaga, 1997). E has the same value for a tension or compression, being a constant as long as the force does not exceed a maximum value called elastic limit (Hooke's law). The formula for calculating the elasticity modulus is:

$$E = \frac{\sigma(\varepsilon)}{\varepsilon} = \frac{F}{A_0}\frac{L_0}{\Delta L} \tag{14}$$

where E is the modulus of elasticity, F is the force, A_0 is the surface (area) where the force is applied, ΔL is the length variation and L_0 is the initial length. Typical values of E for some materials are shown in Table 2.

Table 2: Typical values for E (Larock et al., 2000)

Tabela 2. Typowe wartości E – modułu elastyczności

Material	E, Pa
Steel	$2.077 \cdot 10^{11}$
Copper	$1.1 \cdot 10^{11}$
Bronze	$1.0 \cdot 10^{11}$
Asbestos cement	$2.3 \cdot 10^{10}$
Fiberglass reinforced	$9.0 \cdot 10^{9}$
PVC	$2.8 \cdot 10^{9}$
Polyethylene	$8.0 \cdot 10^{8}$

When a sample of material is stretched in one direction it tends to get thinner in the other two directions (Figure 2). The Poisson's ratio is the ratio of the relative contraction strain (or transverse strain) normal to the applied load. It can be expressed as:

$$u = \frac{-\varepsilon_t}{\varepsilon_L} \tag{15}$$

where u is the Poisson's ratio, εt is the transverse strain and εL is the longitudinal or axial strain. Strain can be expressed as:

$$\varepsilon = \frac{dL}{L} \tag{16}$$

where dL is the change in length and L is the initial length.

For isotropic materials the Poisson's ratio is in the range of 0 to 0.5 (Greaves et al., 2011). Table 3 shows some typical values of u.

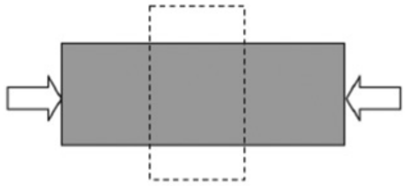


Fig.2: Contraction strain normal to the applied load

Rys.2: Odkształcenie skurczowe normalne do przyłożonego obciążenia

Table 3: Typical values for Poisson's ratio (Larock et al., 2000)

Tabela 3: Typowe wartości współczynnika Poissona

Material	u	Material	u
Steel	0.30	Fiberglass reinforced plastic	0.22
Copper	0.36	PVC	0.45
Bronze	0.34	Polyethylene	0.46
Asbestos cement	0.30		

Conclusions

The MWSA distorts the physical characteristics of the water hammer problem. Due to this, it is recommendable that in the process of discretization (Δx, Δt) of the pipe network, necessary to solve the water hammer in pipe networks by MOC, before deciding to apply the MWSA to obtain Cn = 1, the analyst must see if the final values adopted to calculate a are consistent and appropriate, both in numerical and physical terms. Otherwise it would solve a very different problem originally raised with implications for all stages of design or verification of the system. Before changing the value of a, it is important to check the implications of changing its magnitude. At this point, it is important to know what parameters of its formulation are known and can be considered as unalterable (pipe length, diameter or wall thickness) and check what of the other parameters can be modified by analyzing its variation range and level of reality.

Shock waves for gas pipelines

The fundamental differentia equations describing transient flow in gas pipeline take the form [Osiadacz,1987]:

$$\frac{\partial p}{\partial t} + \alpha_1 \frac{\partial Q_N}{\partial x} = 0 \tag{17}$$

$$\frac{\partial Q_N}{\partial t} = \alpha_2 \frac{\partial p}{\partial x} + \alpha_3 \frac{|Q_N|Q_N}{p} \tag{18}$$

where: $\alpha_1 = \frac{\rho_N a^2}{A}$, $\alpha_2 = \frac{A}{\rho_N}$, $\alpha_3 = \frac{2f\rho_N a}{DA}$

a – velocity of pressure wave propagation [m/s];
A – cross-sectional area of pipe [m^2];
D – internal diameter of pipe [m];
f – friction coefficient;
Q_N – flow under standard conditions ;
L – length of pipe [m];
p – pressure [Pa];
t – time [s];
x – abscissa along the pipeline [m];
p – density of the liquid [kg/m^3].

The equations (17) and (18) were solved using MOC. The original set of partial differentia equations was replaced by two pair of ordinary differentia equations:

$$dx - a\,dt = 0$$

$$-\alpha_2\, dp + \alpha_1 \alpha_2 \frac{dQ_N}{a} - \alpha_3 \frac{|Q_N|}{p} Q_N dx = 0 \quad \text{along a+} \qquad (20)$$

$$dx + a\,dt = 0$$

$$-\alpha_2\, dp - \alpha_1 \alpha_2 \frac{dQ_N}{a} - \alpha_3 \frac{|Q_N|}{p} Q_N dx = 0 \quad \text{along a-} \qquad (21)$$

Thus for

$$t = t(0, t_{\max})$$

$$x = x(0, L)$$

and $a\frac{\Delta t}{\Delta x} \le 1$

after transformations we obtain:

$$p_{B,j}^{k} - p_{A,j} + \alpha_4\left[Q_{NB,j}^{k} - Q_{NA,j}\right] + \alpha_5\left[\frac{|Q_{NB,j}^{k-1} + Q_{NA,j}|}{p_{B,j}^{k-1} + p_{A,j}}\left(\frac{Q_{NB,j}^{k-1} + Q_{NA,j}}{2}\right)\right] = 0 \qquad (22)$$

$$p_{B,j}^{k} - p_{A,j+1} + \alpha_4\left[Q_{NB,j}^{k} - Q_{NA,j+1}\right] + \alpha_5\left[\frac{|Q_{NB,j}^{k-1} + Q_{NA,j+1}|}{p_{B,j}^{k-1} + p_{A,j+1}}\left(\frac{Q_{NB,j}^{k-1} + Q_{NA,j+1}}{2}\right)\right] = 0 \qquad (23)$$

where: $\alpha_4 = \frac{\alpha_1}{a}$, $\alpha_5 = \frac{\alpha_3}{\alpha_1}$

j = 0,1,2,……,J-1

The grid of characteristics is shown in Fig.3
where: p_A – pressure at level A, Q_{NA} – flow at level A
p_B – pressure at level B, Q_{NB} – flow at level B

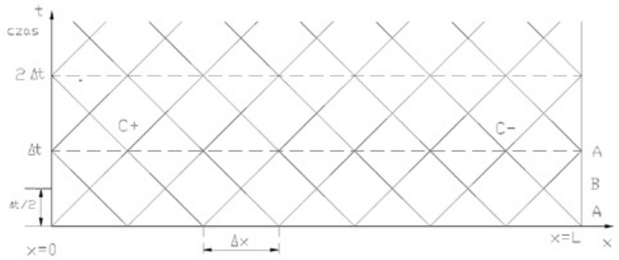


Fig.3 The characteristic grid for a single pipe
Rys.3: Siatka charakterystyk dla pojedyńczego rurociągu

Pressure and flow at level B are calculated using eqs. 24 and 25.

$$p_{B,j}^{k} = 0,5\Bigg(p_{A,j} + p_{A,j+1} + \alpha_4\left[Q_{NA,j+1}\right] + 0,5\alpha_5\left[\frac{|Q_{NB,j}^{k-1} + Q_{NA,j+1}|}{p_{B,j}^{k-1} + p_{A,j+1}}\left(Q_{NB,j}^{k-1} + Q_{NA,j+1}\right) + \frac{|Q_{NB,j}^{k-1} + Q_{NA,j}|}{p_{B,j}^{k-1} + p_{A,j}}\left(Q_{NB,j}^{k-1} + Q_{NA,j}\right)\right]\Bigg) \qquad (24)$$

$$Q_{NB,j}^{k} = Q_{NA,j+1} + \frac{1}{\alpha_4}\left[p_{B,j}^{k} - p_{A,j+1} - 0,5\alpha_5\frac{|Q_{NB,j}^{k-1} + Q_{NA,j+1}|}{p_{B,j}^{k-1} + p_{A,j+1}}\left(Q_{NB,j}^{k-1} + Q_{NA,j+1}\right)\right] \qquad (25)$$

Pressure and flow at level A for j=0 :

$$p_{A,0}^{k} = p_{B,0} = \alpha_4\left[Q_{NA,0}^{k} - Q_{NB,0}\right] + \alpha_5\left[\frac{|Q_{NA,0}^{k-1} + Q_{NB,0}|}{p_{A,0}^{k-1} + p_{B,0}}\left(\frac{Q_{NA,0}^{k-1} + Q_{NB,0}}{2}\right)\right] \qquad (26)$$

$$Q_{NA,0}^{k} = Q_{NB,0} + \frac{1}{\alpha_4}\left[p_{A,0} - p_{B,0} - 0,5\alpha_5\frac{|Q_{NA,0}^{k-1} + Q_{NB,0}|}{p_{A,0} + p_{B,0}}\left(Q_{NA,0}^{k-1} + Q_{NB,0}\right)\right] \qquad (27)$$

For j=J

$$p_{A,J}^{k} = p_{B,J-1} = \alpha_4\left[Q_{NA,J}^{k} - Q_{NB,J-1}\right] + \alpha_5\left[\frac{|Q_{NA,J}^{k-1} + Q_{NB,J-1}|}{p_{A,J}^{k-1} + p_{B,J-1}}\left(\frac{Q_{NA,J}^{k-1} + Q_{NB,J-1}}{2}\right)\right] \qquad (28)$$

$$Q_{NB,J}^{k} = Q_{NA,J-1} + \frac{1}{\alpha_4}\left[p_{A,J}^{k} - p_{B,J-1} - 0,5\alpha_5\frac{|Q_{NA,J}^{k-1} + Q_{NB,J-1}|}{p_{A,J}^{k-1} + p_{B,J-1}}\left(Q_{NA,J}^{k-1} + Q_{NB,J-1}\right)\right] \qquad (29)$$

Pressure and flow for j = 1,2, …. J-1:

$$p_{A,j}^{k} = 0,5\Bigg(p_{B,j} + p_{B,j+1} + \alpha_4\left[Q_{NB,j-1}\right] + 0,5\alpha_5\left[\frac{|Q_{NA,j}^{k-1} + Q_{NB,j-1}|}{p_{A,j}^{k-1} + p_{B,j+1}}\left(Q_{NA,j}^{k-1} + Q_{NB,j}\right) + \frac{|Q_{NA,j}^{k-1} + Q_{NB,j-1}|}{p_{A,j}^{k-1} + p_{B,j} - 1}\left(Q_{NA,j}^{k-1} + Q_{NB,j-1}\right)\right]\Bigg) \qquad (30)$$

$$Q_{NA,j}^{k} = Q_{NB,j} + \frac{1}{\alpha_4}\left[p_{A,j} - p_{B,j} - 0,5\alpha_5\frac{|Q_{NA,j}^{k-1} + Q_{NB,j}|}{p_{A,j} + p_{B,j}}\left(Q_{NA,j}^{k-1} + Q_{NB,j}\right)\right] \qquad (31)$$

Shock waves for two phases flow

Fluid hammer is a pressure surge or wave that occurs when a fluid (usually a liquid, but sometimes a gas) in motion is forced to stop or change direction suddenly (i.e., momentum change). This phenomenon commonly occurs when a valve is closed unexpectedly at the end of a pipeline system and a pressure wave propagates in the flowline. It may also be known as hydraulic shock. This pressure wave can cause major problems, from noise and vibration to flowline rupture. If the pipeline is closed swiftly at the outlet (downstream), the mass of fluid before the closure is still moving forward with a certain velocity, building up high pressure and shock waves.

These may cause a loud bang or repetitive banging (as the shock waves travel back and forth), which could cause pipeline rupture.

On the other hand, when an upstream valve in a pipeline is closed, the fluid downstream of the valve will attempt to continue flowing, creating a vacuum that may cause the pipe to collapse or implode. This problem can be particularly acute if the pipeline is on a downhill slope.

In the field, wellhead and bottomhole pressures do not build up or deplete smoothly after well shut-in or startup, respectively. The resultant erratic pressure fluctuations are common, especially in offshore operations in which packers are commonly installed. The cyclic pressure surge introduced by a sudden momentum change may unset packers, hammer tubing, and damage the well completion, and may cause sand-control and other flow-assurance issues that could be very costly. Therefore, this transient scenario has drawn significant attention from the industry in the past few decades, and many independent studies have been performed in this area.

Transporting of natural gas in pipelines can take place in any of four different modes shown in fig.4.

- Case 1 is one in which single-phase gas flow exists throughout the pipe. In this case, no condensation occurs, hence no liquid holdup prediction is necessitated. In such a case, the single-phase flow model is used. In fact, any of the popular gas flow design equations such as Weymouth and Panhandle equations is often used for practical design of this type of system.
- Case 2 is the one in which the flow is totally in the two-phase region. In this case, a two-fluid model can be applied for predicting pressure, gas velocity, liquid velocity and liquid holdup profiles in a pipeline.
- Case 3 , the inlet is in the single-phase conditions and after a certain distance, the fluid enters the two-phase region due to pressure and temperature changes.
- Case 4 is the one in which the inlet conditions fall into the two=-phase region but the downstream conditions fall in the single--phase region. In practice, none of these casusu is known beforehand and hence a predicting model is necessary.

The theory of water hammer for an ideal fluid was created by N. E Zhukovsky [1]. This theory is based on the assumption that the fluid moving in the pipeline is a continuous homogeneous medium (the so-called "pure" or "single-phase" fluid). Obviously, real liquids operating in irrigation systems, drainage of water supply, etc., differ to one degree or another from a "pure" liquid, since they always contain a certain amount of gaseous and solid impurities.

The presence of impurities significantly affects the nature of water hammer in pressure pipelines. Therefore, the calculations of the water hammer of a real fluid based on a homogeneous model do not have sufficient accuracy in many cases. To increase the accuracy of calculations, it becomes necessary to take into account the complex phase structure of real liquids, i.e., to construct calculated dependences based on models of multiphase media. Of particular interest is the study of a two-phase (water + air) gas-liquid flow, since, firstly, liquids in irrigation networks almost always contain not dissolved gases. Secondly, it is the presence of gas inclusions that has the strongest effect on the rate of propagation of disturbances in the medium-one of the most important factors determining the entire course of the no stationary process.

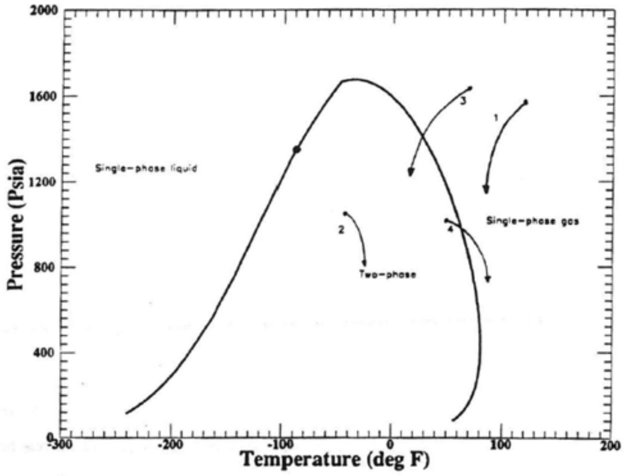


Fig.4 Flow scenarios in gas pipelines

Rys.4: Charakter przepływu w gazociągach

The speed of propagation of the shock wave of the water hammer is the main parameter when calculating the water hammer.

The speed of propagation of a shock wave in an unlimited gas-liquid flow is determined by the ratio

$$c = \sqrt{\frac{p}{(1-\varepsilon)\varepsilon\rho_{ж}}}, \tag{32}$$

Here ρ_* is the density of the liquid; ε is the volumetric content of not dissolved gases in the mixture. Equation (32) called the low-frequency approximation of the shock wave velocity is derived under assumptions of the incompressibility of the liquid and the isothermal law of compression-expansion of the gas. Introducing into consideration the shock wave velocity in an unlimited volume of a "pure" liquid

$$c_{ж} = \sqrt{\frac{E_{ж}}{\rho_{ж}}} \tag{33}$$

here : Eж is the bulk modulus of elasticity of the liquid, and generalizing to the case of polytropic behaviour of the gas in bubble we get:

$$c = \frac{\sqrt{\frac{E_{ж}}{\rho_{ж}}}}{\sqrt{\varepsilon\frac{E_{ж}}{n\cdot p}}} = \frac{c_{ж}}{\sqrt{\varepsilon\frac{E_{ж}}{n\cdot p}}}, \tag{34}$$

where: n is the exponent of the polytropic degree.

The formula for calculating the speed of sound in a two-phase flow taking into account the compressibility of the liquid phase is the following:

$$c = \sqrt{\frac{E_{ж}}{\rho(1-\varepsilon)\left(1+\varepsilon\frac{E_{ж}}{n\cdot p}\right)}}. \tag{35}$$

In the case when the process of compression-expansion of the gas phase is assumed to be isothermal, the following formula to calculate the velocity of a shock wave in a two-phase flow is given.

$$c = \frac{1}{\sqrt{\rho_{ж}\left(\frac{1}{E_{ж}}+\frac{\varepsilon}{p}+\frac{d}{p\cdot e}\right)}}. \tag{36}$$

where: d – the inner pipe diameter, e – wall thickness

Simulation of multiphase fluid hammer effects

The fully transient software have the capacity to model multiphase flow in wellbore and pipeline by solving five coupled mass-conservation equations, three momentum-conservation equations, and one energy balance equation for a three phase system (Bendiksen et al. 1991).

Mass-Conservation Equations. For gas phase,

$$\frac{\partial}{\partial t}(V_g\rho_g) = -\frac{1}{A}\frac{\partial(AV_g\rho_g v_g)}{\partial z} + \psi_g + G_g. \tag{37}$$

For liquid phase at pipe wall,

$$\frac{\partial}{\partial t}(V_L\rho_L) = -\frac{1}{A}\frac{\partial}{\partial z}(AV_L\rho_L v_L) - \psi_g\frac{V_L}{V_L+V_D} - \psi_e + \psi_d + G_L. \tag{38}$$

For liquid droplets,

$$\frac{\partial}{\partial t}(V_D\rho_L) = -\frac{1}{A}\frac{\partial(AV_D\rho_L v_D)}{\partial z} - \psi_g\frac{V_D}{V_L+V_D} + \psi_e - \psi_d + G_D. \tag{39}$$

For phase transfer between phases,

$$\left[\frac{V_g}{\rho_g}\left(\frac{\partial\rho_g}{\partial p}\right)_{T,R_s} + \frac{1-V_g}{\rho_L}\left(\frac{\partial\rho_L}{\partial p}\right)_{T,R_s}\right]\frac{\partial p}{\partial t}$$
$$= -\frac{1}{A\rho_g}\frac{\partial(AV_g\rho_g v_g)}{\partial z} - \frac{1}{A\rho_L}\frac{\partial(AV_L\rho_L v_L)}{\partial z}$$
$$-\frac{1}{A\rho_L}\frac{\partial(AV_D\rho_L v_D)}{\partial z} + \psi_g\left(\frac{1}{\rho_g}-\frac{1}{\rho_L}\right)$$
$$+G_g\frac{1}{\rho_g} + G_L\frac{1}{\rho_L} + G_D\frac{1}{\rho_L} \tag{40}$$

For interfacial mass-transfer rate,

$$\psi_g = \left[\left(\frac{\partial R_s}{\partial p}\right)_T\frac{\partial p}{\partial t} + \left(\frac{\partial R_s}{\partial p}\right)_T\frac{\partial p}{\partial z}\frac{\partial z}{\partial t} + \left(\frac{\partial R_s}{\partial T}\right)_p\frac{\partial T}{\partial t} + \left(\frac{\partial R_s}{\partial T}\right)_p\frac{\partial T}{\partial z}\frac{\partial z}{\partial t}\right]$$
$$\times(m_g + m_L + m_D), \tag{41}$$

where

$$R_s = \frac{m_g}{m_g + m_L + m_L}. \tag{42}$$

Momentum-Conservation Equations. For gas phase,

$$\frac{\partial(V_g\rho_g v_g)}{\partial t}=-V_g\left(\frac{\partial p}{\partial z}\right)-\frac{1}{A}\frac{\partial(AV_g\rho_g v_g^2)}{\partial z}$$
$$-\lambda_g\frac{1}{2}\rho_g|v_g|v_g\frac{S_g}{4A}-\lambda_i\frac{1}{2}\rho_g|v_r|v_r\frac{S_i}{4A}+V_g\rho_g g\cos\alpha+\psi_g v_a-F_D. \tag{43}$$

For liquid phase at pipe wall,

$$\frac{\partial(V_L\rho_L v_L)}{\partial t}=-V_L\left(\frac{\partial p}{\partial z}\right)-\frac{1}{A}\frac{\partial(AV_L\rho_L v_L^2)}{\partial z}$$
$$-\lambda_L\frac{1}{2}\rho_L|v_L|v_L\frac{S_L}{4A}+\lambda_i\frac{1}{2}\rho_g|v_r|v_r\frac{S_i}{4A}+V_L\rho_L g\cos\alpha$$
$$-\psi_g\frac{V_L}{V_L+V_D}v_a-\psi_e v_i+\psi_d v_D-V_L d(\rho_L-\rho_g)g\frac{\partial V_L}{\partial z}\sin\alpha\,. \tag{44}$$

For liquid droplets,

$$\frac{\partial(V_D\rho_L v_D)}{\partial t}=-V_D\left(\frac{\partial p}{\partial z}\right)-\frac{1}{A}\frac{\partial(AV_D\rho_L v_D^2)}{\partial z}$$
$$+V_D\rho_L g\cos\alpha-\psi_g\frac{V_D}{V_L+V_D}v_a+\psi_e v_i-\psi_d v_D+F_D, \tag{45}$$

where $v_a=v_L$ for $\psi_g>0$ (and evaporation from the liquid film), $v_a=v_D$ for $\psi_g>0$ (and evaporation from the liquid droplets), and $v_a=v_g$ for $\psi_g<0$ (condensation).

Mixture Energy-Conservation Equation.

$$\frac{\partial}{\partial t}\left[m_g\left(E_g+\frac{1}{2}v_g^2+gh\right)+m_L\left(E_L+\frac{1}{2}v_L^2+gh\right)+m_D\left(E_D+\frac{1}{2}v_D^2+gh\right)\right]$$
$$=\frac{\partial}{\partial z}\left[m_g v_g\left(H_g+\frac{1}{2}v_g^2+gh\right)+m_L v_L\left(H_L+\frac{1}{2}v_L^2+gh\right)\right.$$
$$\left.+m_D v_D\left(H_D+\frac{1}{2}v_D^2+gh\right)\right]+H_S+U\,. \tag{46}$$

To solve above set of partial differentia equations, implicit numerical schemes are the most effective. Sometimes (Chaudhury, 1994) explicit methods yield satisfactory results with smaller time step (stability and accuracy) but smaller time step will increase computation time.

A typical offshore production scheme as shown in Fig.5 is used to verify quality of the software. It consists of a topside platform, riser, flowline, wellbore and well completion.

Conclusions

Software that is capable of modelling fully transient multiphase flow in wellbore and pipeline to characterize the fluid hammer effects is necessary for operator a offshore production system. Field operations, such as pressure transient analysis, facility maintenance, and workover, require a well shut-in process. For a typical production system, the resultant sudden rises in pressure can be critical because they have a direct impact on equipment and may cause damage to instrumentation. Using software it is possible to provide estimates of the typical ratio of transient shock in pressure and flow rate to preconditional values, and the duration of such pressure shocks. It also can propose the best location for the shut-in valve and the length of flowline needed to reduce the fluid-hammer effects.

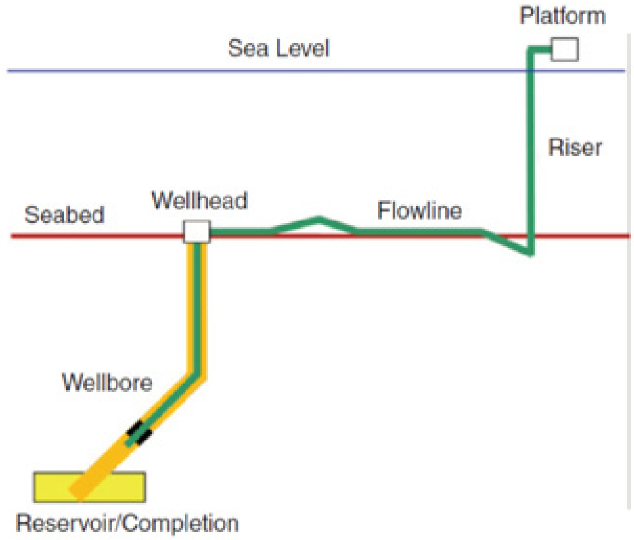


Fig.5: A offshore production system

Rys.5: System podmorskiej eksploatacji

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